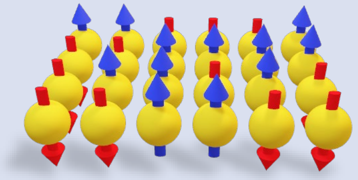
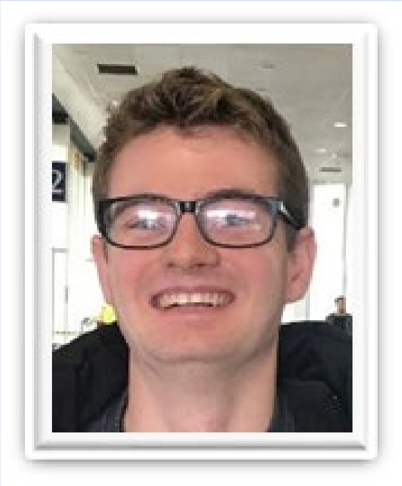
Dissipation induced non-stationary complex quantum dynamics

Dieter Jaksch, Clarendon Laboratory, University of Oxford



4 February 2021

Collaborators



Joey Tindall
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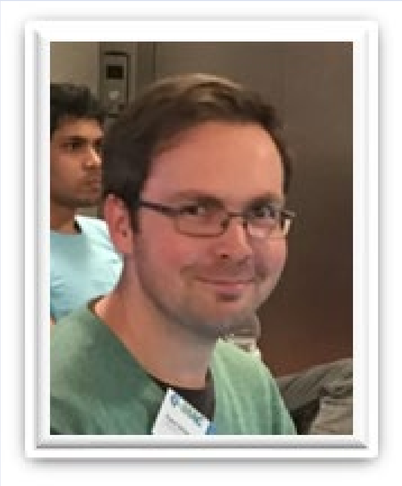

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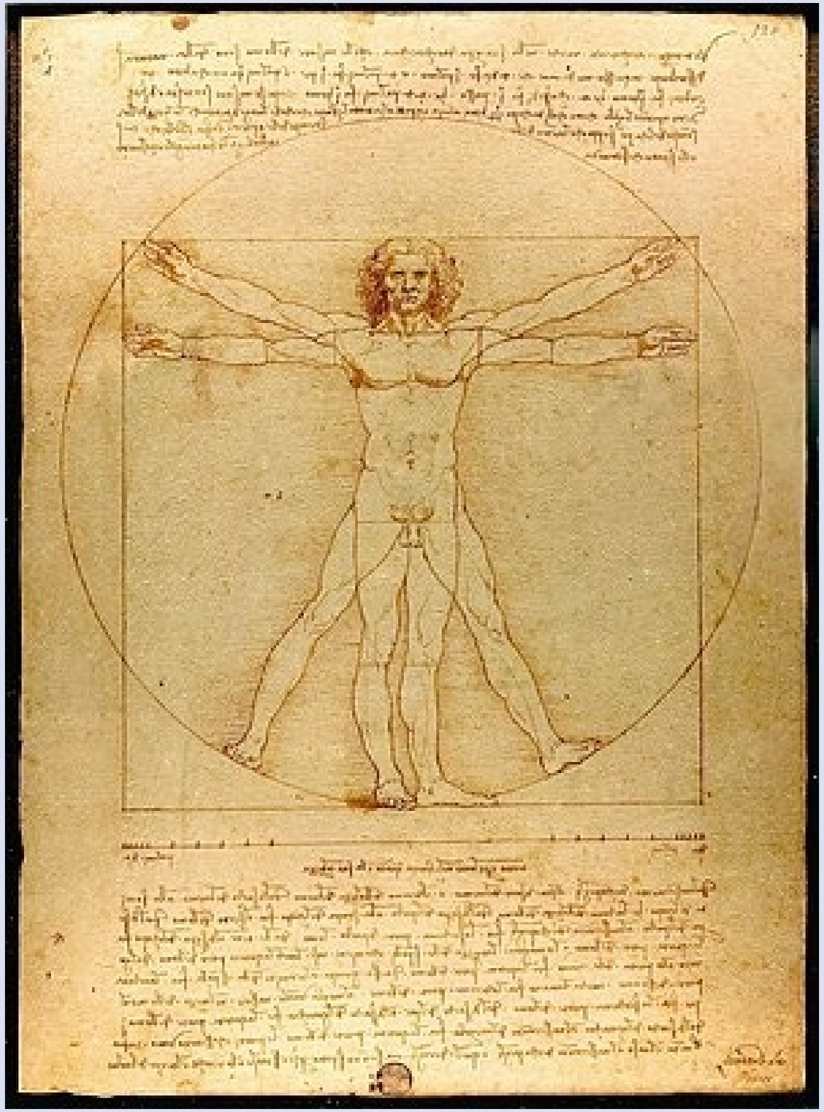
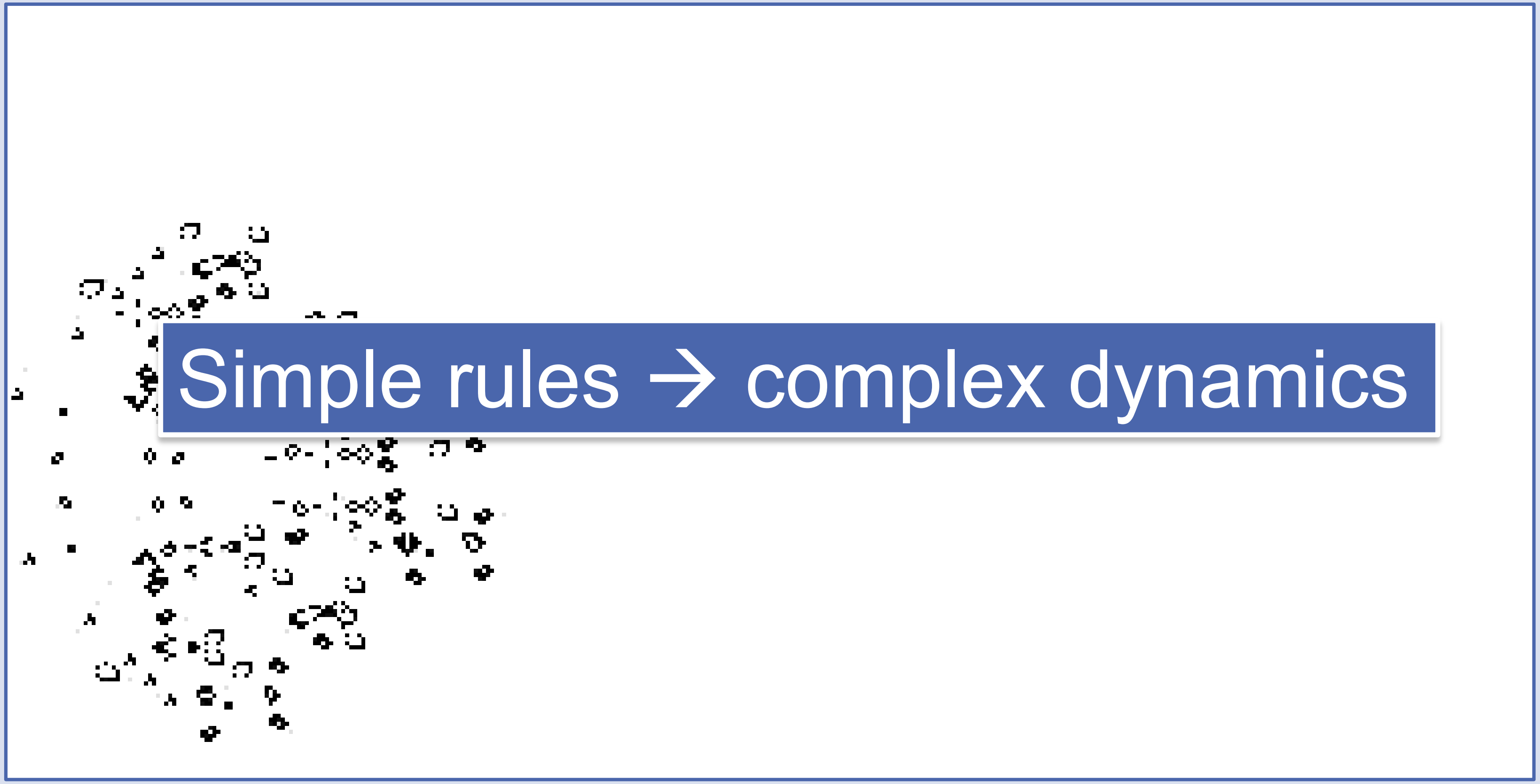
- *Complex coherent quantum many-body dynamics through dissipation,* B. Buca, J. Tindall, D. Jaksch, Nature Communications **10**, 1730 (2019).
- *Heat-induced Long-Range η-Pairing in the Hubbard Model, J. Tindall, B. Buca, J. R. Coulthard, D. Jaksch, Physical Review Letters **123**, 030603 (2019).*
- *Dynamical Order and Superconductivity in a Frustrated Many-Body System,* J. Tindall, F. Schlawin, M. Buzzi, D. Nicoletti, J.R. Coulthard, H. Gao, A. Cavalleri, M. Sentef and D. Jaksch, Phys. Rev. Lett. **125**, 137001 (2020).
- *Analytical Solution for the Steady States of the Driven Hubbard model,* J. Tindall, F. Schlawin, M. Sentef and D. Jaksch, Phys. Rev. B **103**, 035146 (2021).

Complex systems – no stationarity



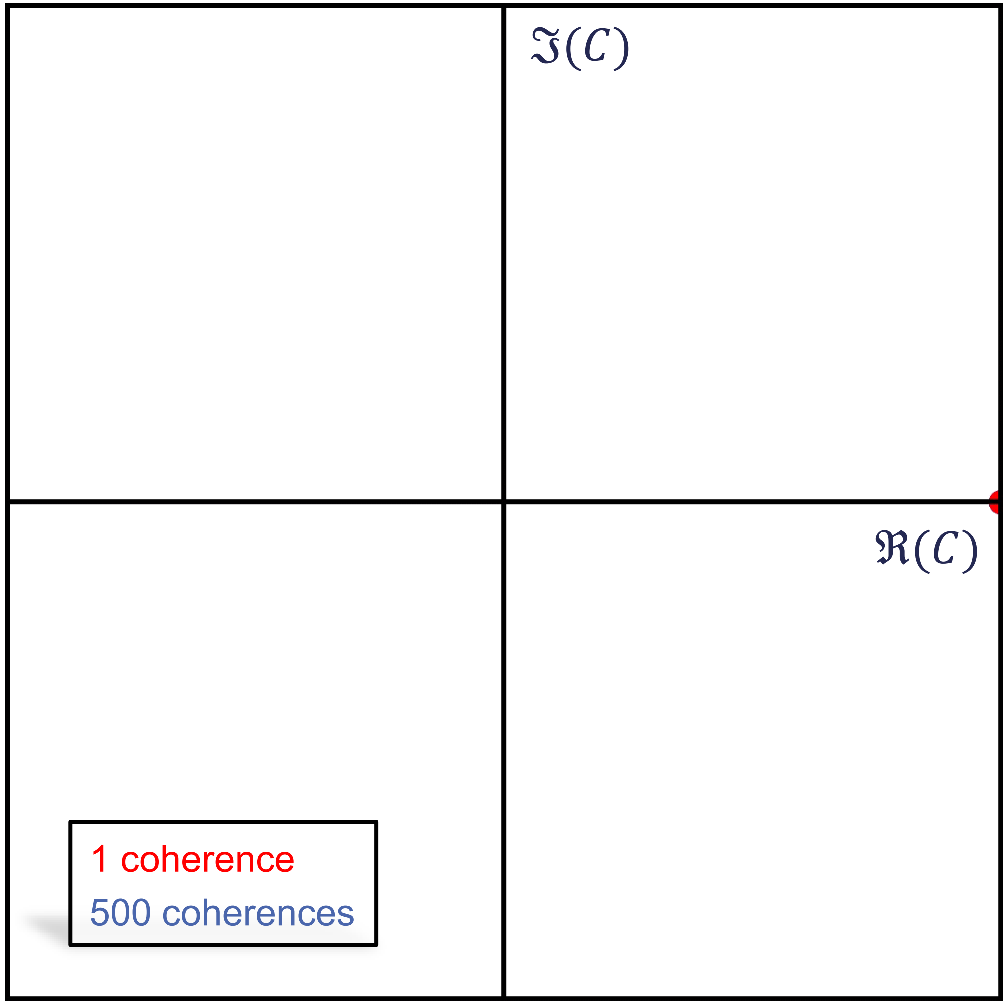
Conway’s game of life



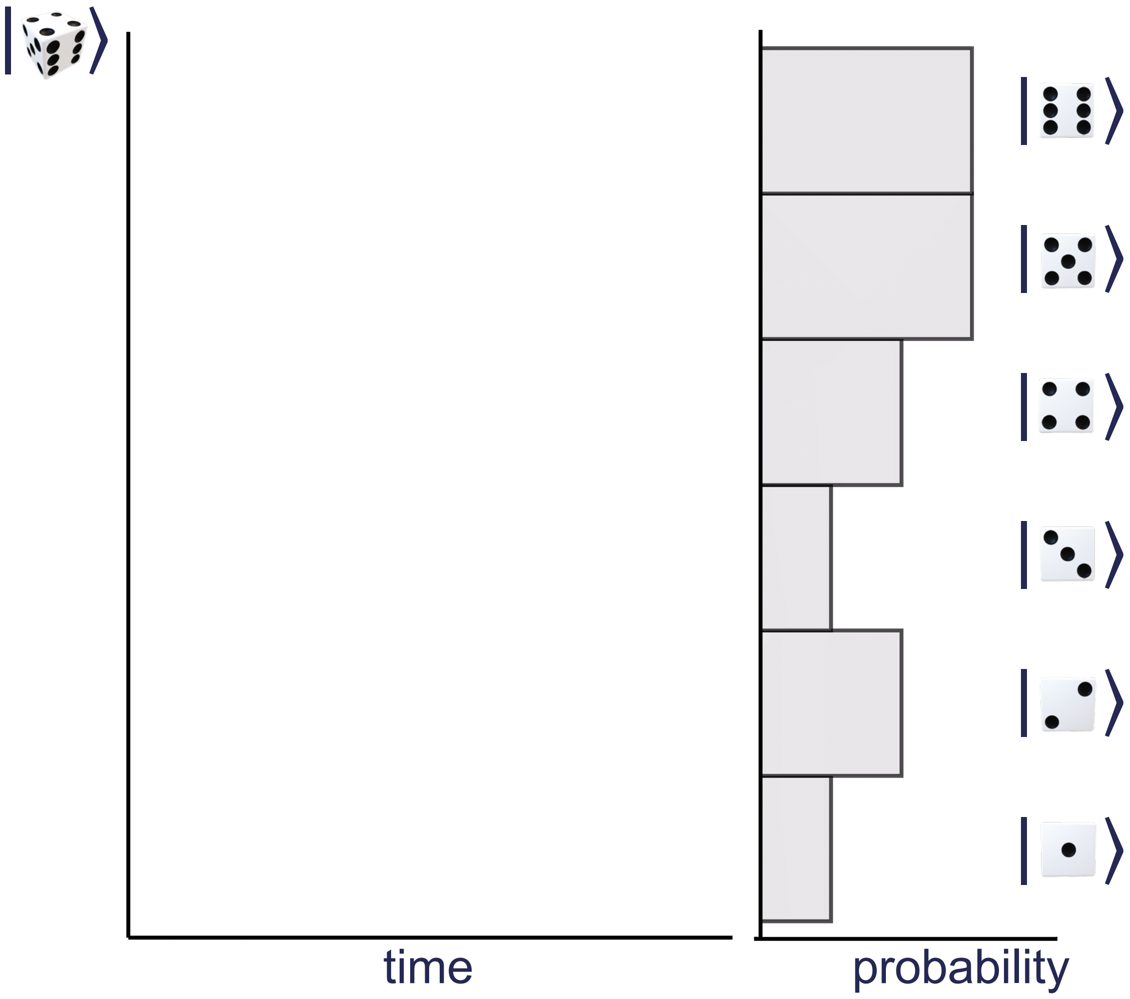
Obtained from: http://web.stanford.edu/~cdebs/GameOfLife/

Quantum systems – ETH and ergodicity

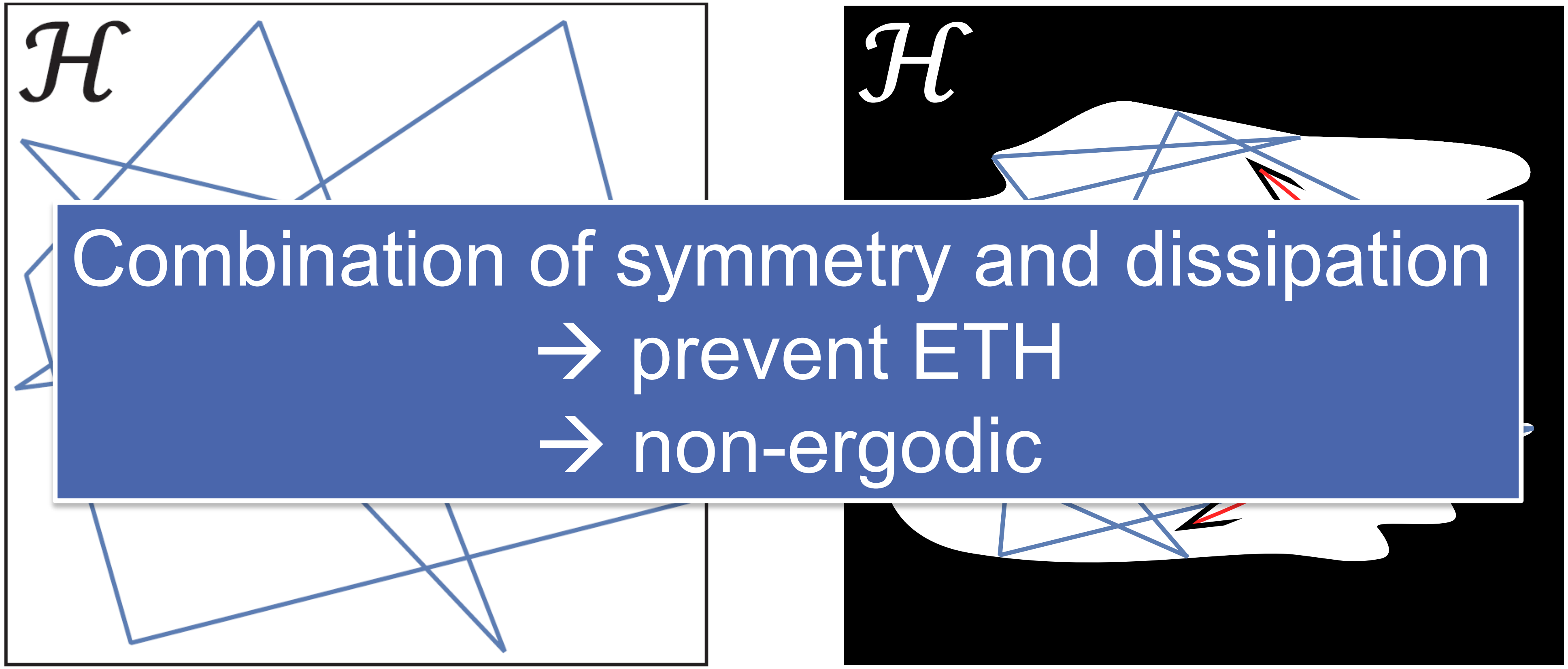
Closed system – dephasing $C = \sum_{n \neq m} \langle n | \hat{C} | m \rangle$



Open system – ergodicity



Non-stationary quantum matter



Combination of symmetry and dissipation
→ prevent ETH
→ non-ergodic

Can we engineer a many-body quantum system that shows “perpetual” non-stationary dynamics?

Eigenvalues λ_i and eigenstates ρ_i of the dynamics

$$\rho_i = e^{\lambda_i}\rho_i$$

$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H,\rho]$$

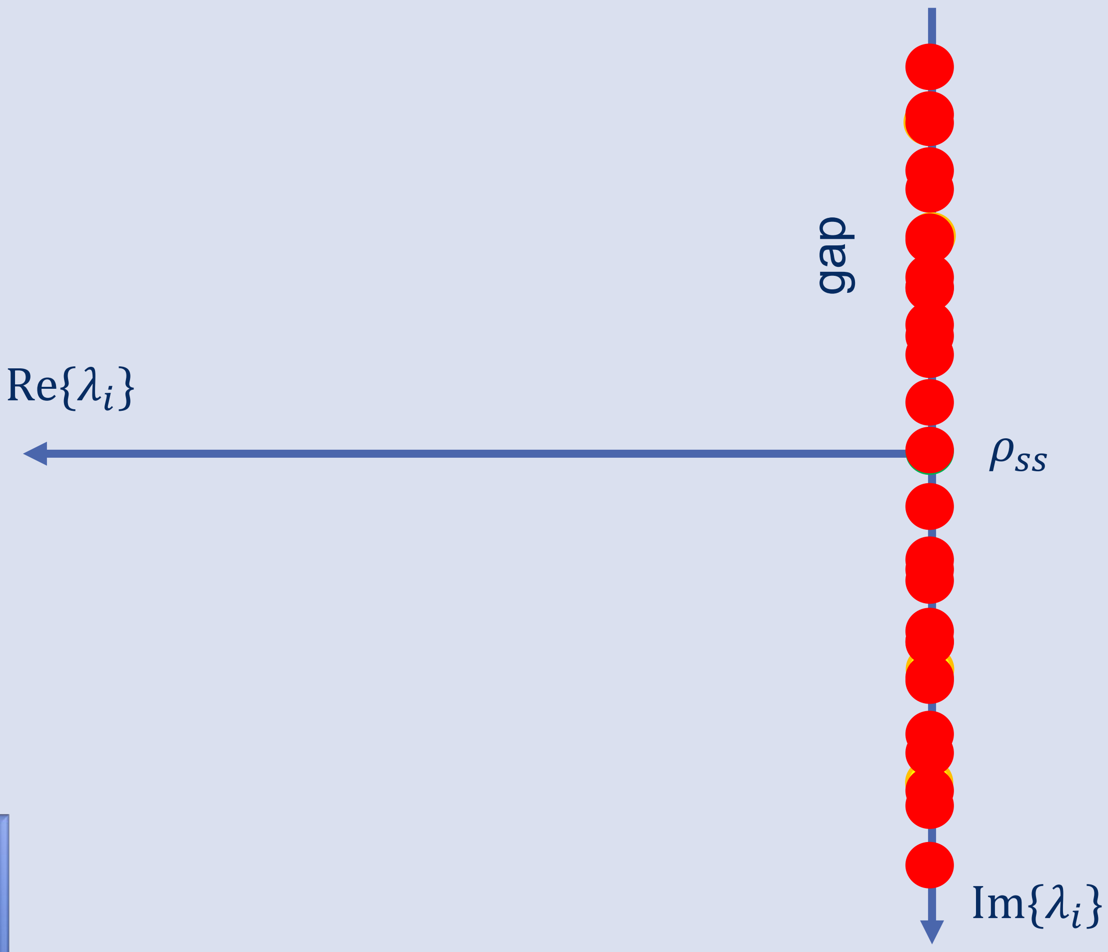
$$+\sum_\mu \left(2\, L_\mu\, \rho\, L_\mu^\dagger - L_\mu^\dagger L_\mu\, \rho - \rho L_\mu^\dagger L_\mu\right)$$

Closed system

- purely imaginary λ_i
- dephasing

Open system

- negative real parts
- dissipation



Generic condition for

$[H, A] = -\lambda A$ and $\left[A, L_\mu^\dagger\right] = [A, L_\mu] = 0 \quad \forall\mu$

Generic condition

- If there exists a lowering operator A with the following properties

$$[H, A] = -\lambda A \quad \text{and} \qquad \left[A, L_\mu^\dagger\right] = \left[A, L_\mu\right] = 0 \quad \forall \mu$$

with $\lambda \neq 0$ then ρ_{nm} of the form (with integer $m, n > 0$, $m \neq n$)

$$\rho_{nm} = \mathrm{A}^n \rho_{00} \left(A^\dagger\right)^m$$

is non-stationary with $\mathcal{L}\rho_{nm} = \mathrm{i}(m-n)\lambda\rho_{nm}$

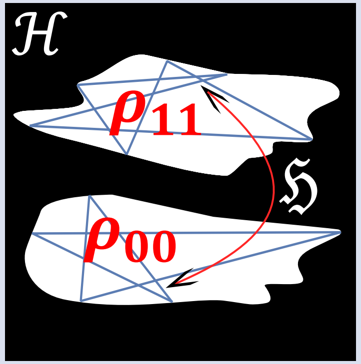
- The mixed coherences are not necessarily decoupled from the environment

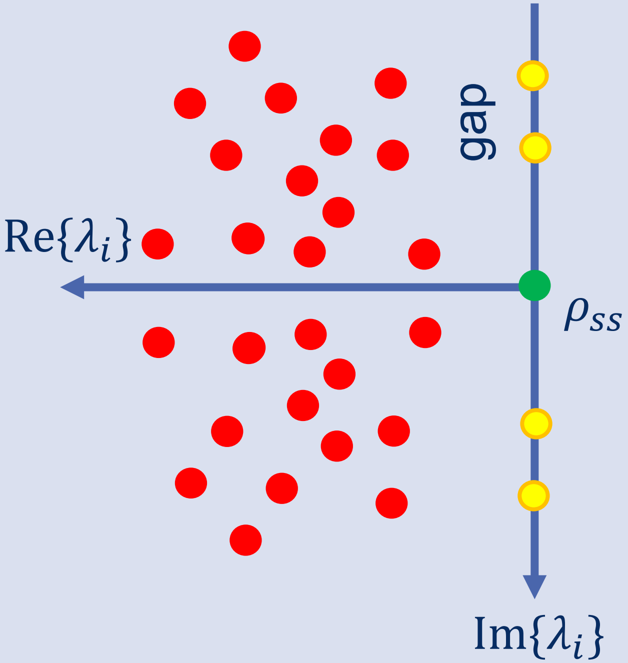
$$L_\mu \rho_{nm} L_\mu^\dagger \neq 0$$

- A dark Hamiltonian $\mathfrak{H}$ describes this long-time evolution

  - $\mathfrak{H}$ is not necessarily Hermitian, eigenstates not pure

  - evolution by $\mathfrak{H}$ not decoupled from the environment

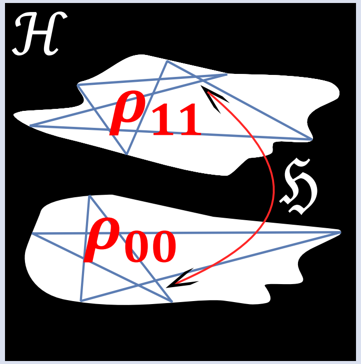




Long-time evolution

- We consider Hermitian quantum jump operators L_μ.
- The identity operator $\mathbb{I}$ is then a stationary state of the dynamics

$$\mathcal{L}\mathbb{I} = -\mathrm{i}[H, \mathbb{I}] + \sum_\mu \left(2\, L_\mu \mathbb{I} L_\mu^\dagger - L_\mu^\dagger L_\mu \mathbb{I} - \mathbb{I} L_\mu^\dagger L_\mu\right) = 0$$

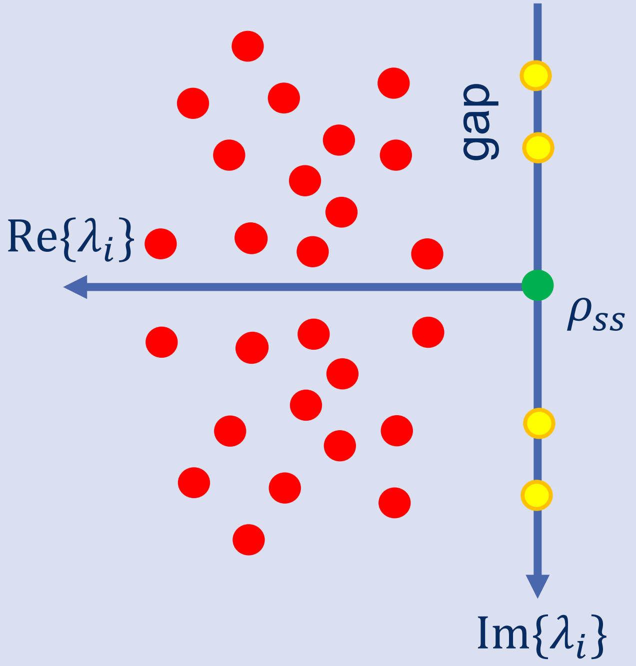


- The stationary state is a function of the conserved quantities only.
- We assume thermalization of the state within each symmetry sector

$$\rho(\tau \to \infty) \propto \exp\left(\sum_i \beta_i C_i\right)$$

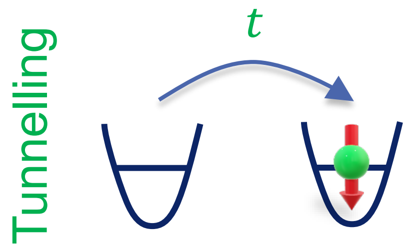
where C_i are the operators describing the conserved quantities.

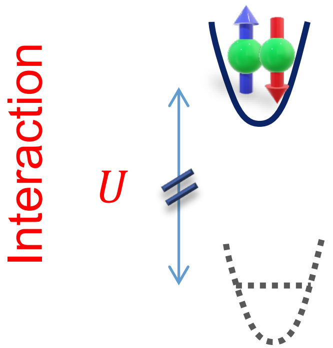
- Their expectation values are fixed by the Lagrange multipliers β_i.
- This is an infinite temperature state if H is not conserved.

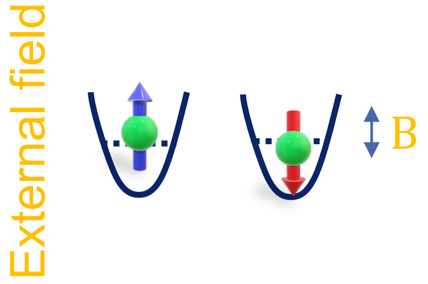


Paradigm example: The Fermi Hubbard model

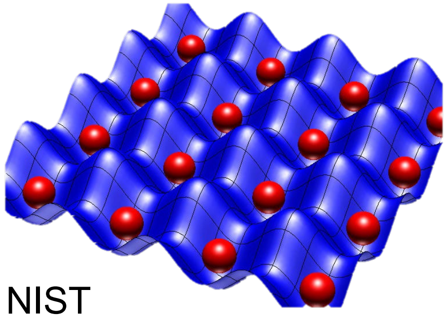
$$H_{\mathrm{Hub}} = -t \sum_{\langle i,j\rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.\right) \quad + \quad U \sum_{i} n_{i\uparrow} n_{i\downarrow} \quad + \quad \frac{B}{2} \sum_{i} n_{i\uparrow} - n_{i\uparrow} \quad - \quad \mu \sum_{i} n_{i}$$







Optical lattices:



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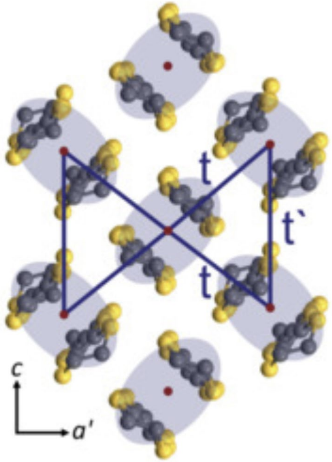
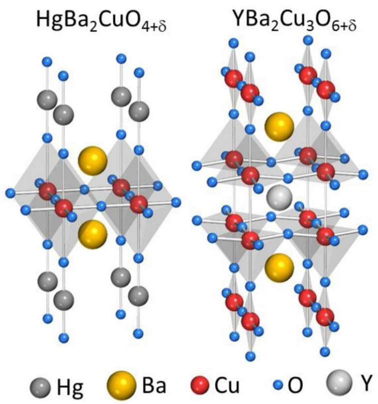
Organic salts:



© Crystals

Cuprates:



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Hubbard model and spin symmetry

- Hubbard Hamiltonian $H_{Hub} = H_{\mathrm{hop}} + H_U + H_\mu + H_B$

$$H_{\mathrm{Hub}} = -t \sum_{\langle i,j\rangle,\sigma} \left(c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + \frac{B}{2} \sum_i n_{i\uparrow} - n_{i\uparrow}$$

- Spin symmetry

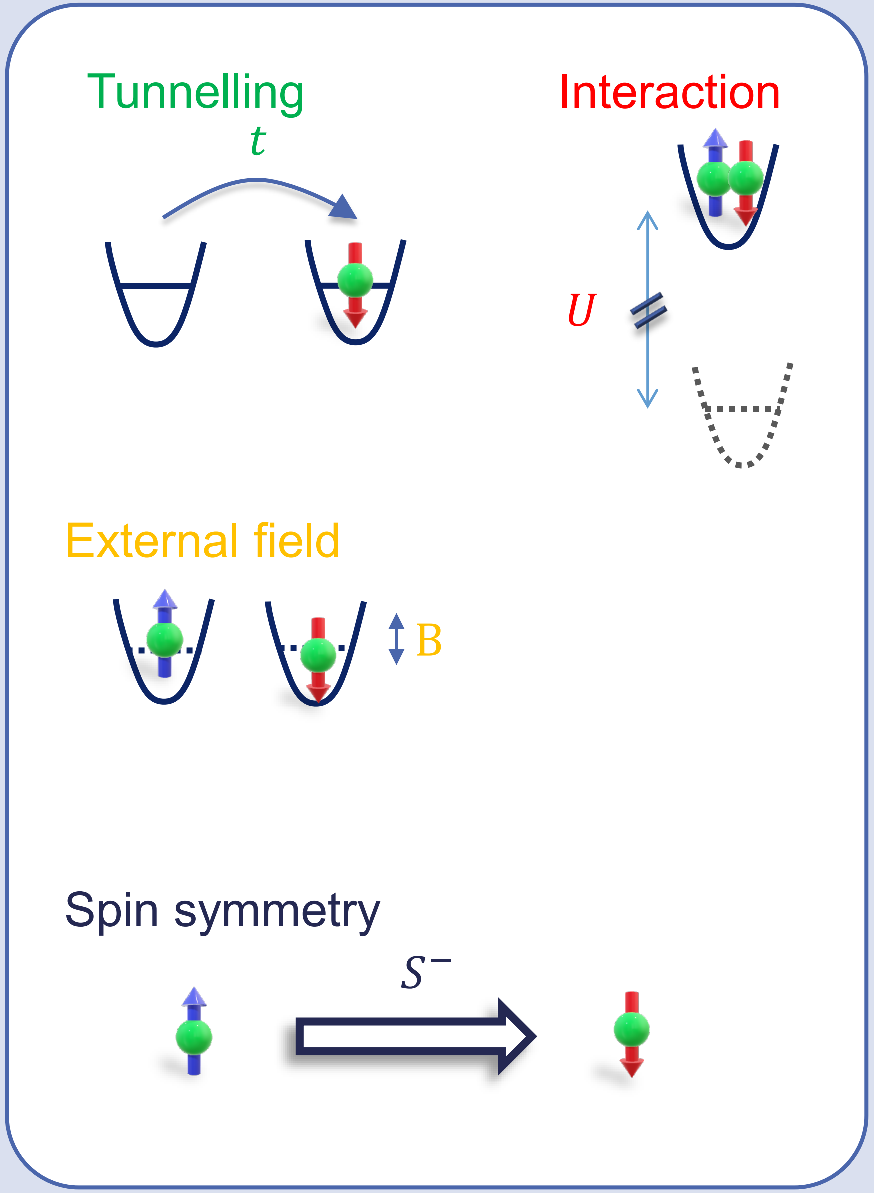
$$S^z = \sum_j S_j^z, \qquad S_j^z = \frac{1}{2}(n_{j,\uparrow} - n_{j,\downarrow}),$$

$$S^+ = \sum_j S_j^+, \qquad S_j^+ = c_{j,\uparrow}^\dagger \, c_{j,\downarrow}$$

$$S^- = \sum_j S_j^-, \qquad S_j^- = c_{j,\downarrow}^\dagger \, c_{j,\uparrow},$$

- These operators fulfil

$$[H_{\mathrm{Hub}}, S^z] = 0, \qquad [H_{\mathrm{Hub}}, S^\pm] = \pm\, B\, S^\pm$$



Hubbard model and η symmetry

- Hubbard Hamiltonian $H_{Hub} = H_{\text{hop}} + H_U + H_\mu + H_B$

$$H_{\text{Hub}} = -t \sum_{\langle i,j \rangle,\sigma} \left(c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + \frac{B}{2} \sum_i n_{i\uparrow} - n_{i\downarrow}$$

- η-pairing symmetry

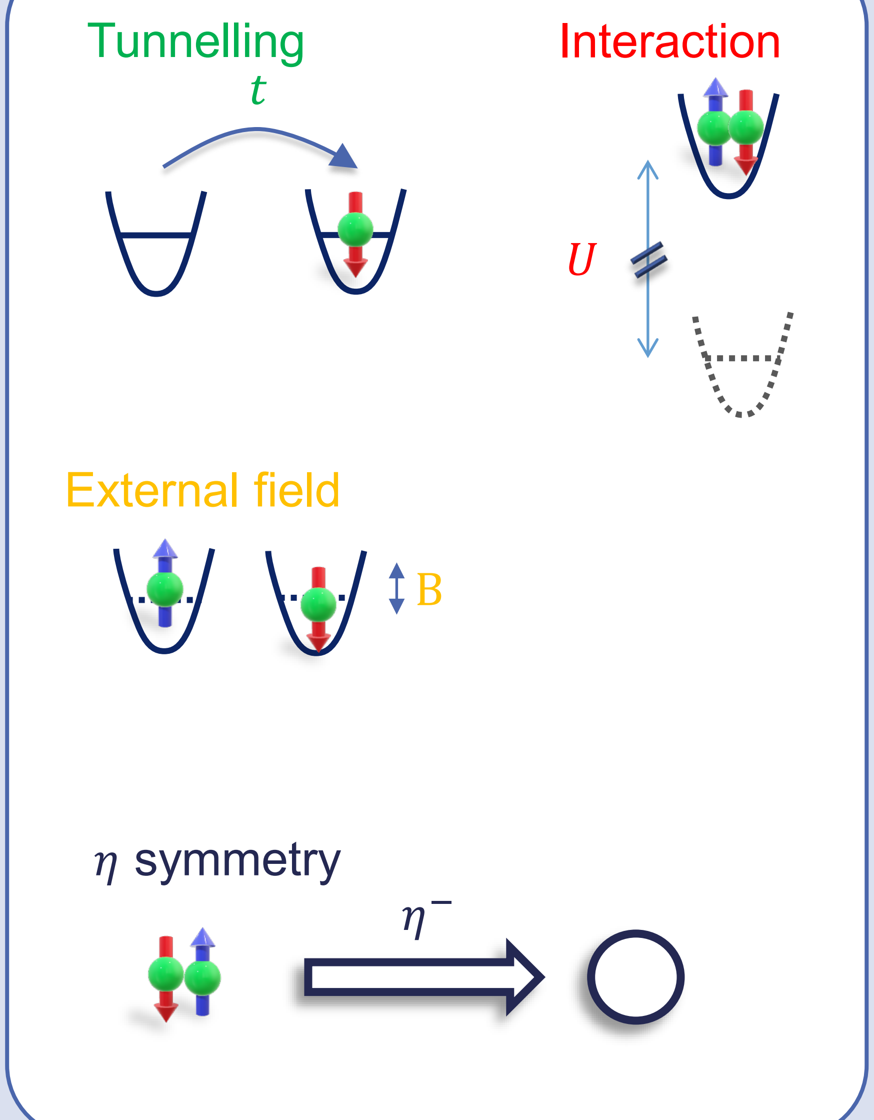
$$\eta^z = \frac{1}{2} \sum_j (n_j - 1) \,,$$

$$\eta^+ = \sum_j \tau(j) \eta_j^+ , \quad \eta_j^+ = c_{j,\uparrow}^\dagger \, c_{j,\downarrow}^\dagger ,$$

$$\eta^- = \sum_j \tau(j) \eta_j^- \,, \qquad \eta_j^- = \; c_{j,\downarrow} \, c_{j,\uparrow} ,$$

with $\tau(j)$ a chequerboard pattern. They fulfil

$$[H_{\text{Hub}}, \eta^z] = 0, \qquad [H_{\text{Hub}}, \eta^\pm] = \pm \, 2\mu\eta^\pm$$



A driven-dissipative Hubbard model

- Hubbard Hamiltonian $H_{Hub} = H_{\text{hop}} + H_U + H_\mu + H_B$

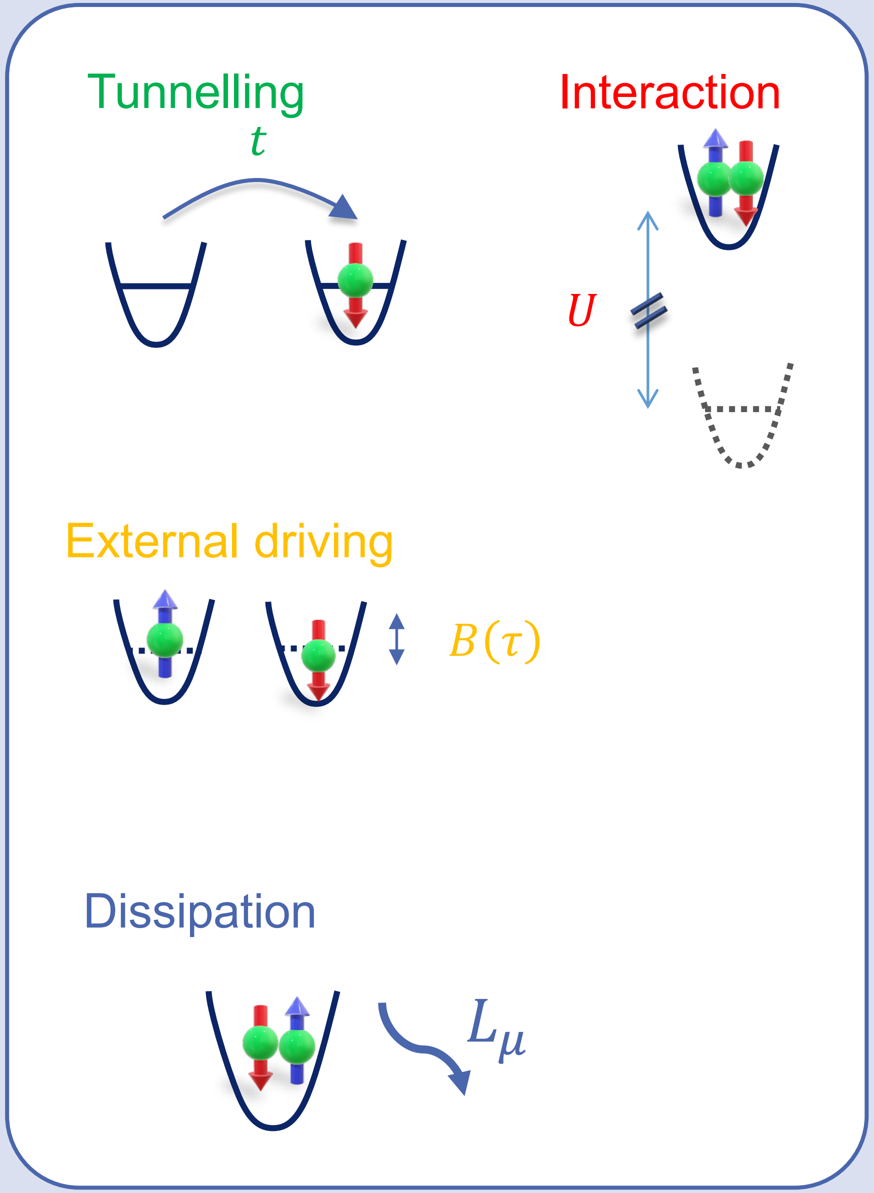
$$H_{\text{Hub}} = -t \sum_{\langle i,j\rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.\right) + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i + \frac{B}{2} \sum_i \; n_{i\uparrow} - n_{i\downarrow}$$

- External driving

$$H_B \rightarrow H_B(\tau) = \frac{1}{2} \sum_i B_i(\tau)(n_{i\uparrow} - n_{i\downarrow})$$

- Dissipation

$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H_{\text{Hub}}, \rho] \; + \sum_\mu \left(2\, L_\mu \,\rho\, L_\mu^{\dagger} - L_\mu^{\dagger} L_\mu \,\rho - \rho L_\mu^{\dagger} L_\mu\right)$$



Spectrum engineering

- Starting from a generic master equation

$$\dot{\rho} = \mathcal{L}\rho = -\mathrm{i}[H_{\mathrm{Hub}}, \rho] + \sum_{\mu} (2\, L_\mu\, \rho\, L_\mu^\dagger - L_\mu^\dagger L_\mu\, \rho - \rho L_\mu^\dagger L_\mu)$$

Dissipation in the spin sector

$$L_\mu \propto S_\mu^z$$

off-diagonal long-range correlations between η pairs

→ **superfluid state**

Dissipation in the charge sector

$$L_\mu \propto n_\mu$$

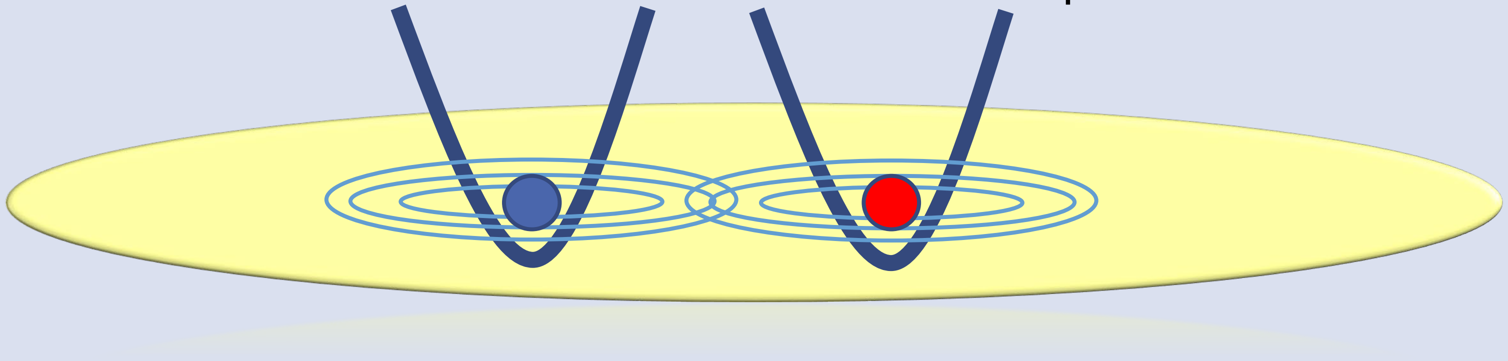
prevent stationarity and eigenstate thermalization

→ **non-stationarity**

B. Buca, J. Tindall, DJ, Nature Comm. **10,** 1730 (2019); J. Tindall, B. Buca, J.R. Coulthard, DJ, PRL 123, 030603 (2019)

Immerse a fermionic lattice into a BEC

Optical lattice = Hubbard model



- The interaction with the BEC atoms will dephase the lattice wave function locally

$$L_\mu \propto a_\uparrow \, n_{\mu\uparrow} + a_\downarrow \, n_{\mu\downarrow}$$

- If $a_\uparrow = a_\downarrow$ then we realize coupling of the bath to the density sector

$$L_\mu \propto n_\mu$$

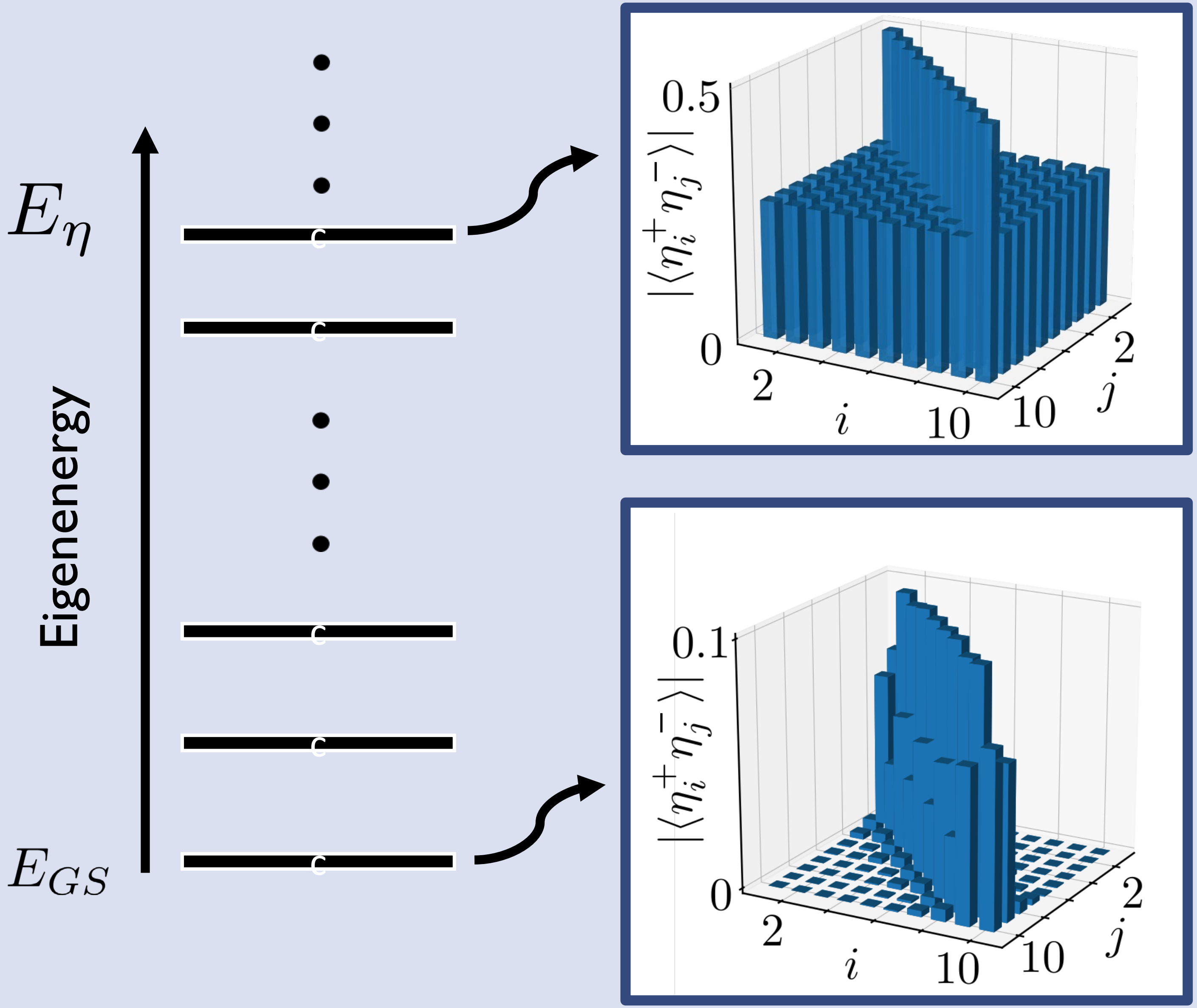
- If $a_\uparrow = -a_\downarrow$ then we realize coupling of the bath to the spin sector

$$L_\mu \propto S_\mu^z$$

M. Bruderer and DJ, New J. Phys. **8**, 87 (2006)

I. Coupling to the Spin Sector

η – correlations in the Hubbard model

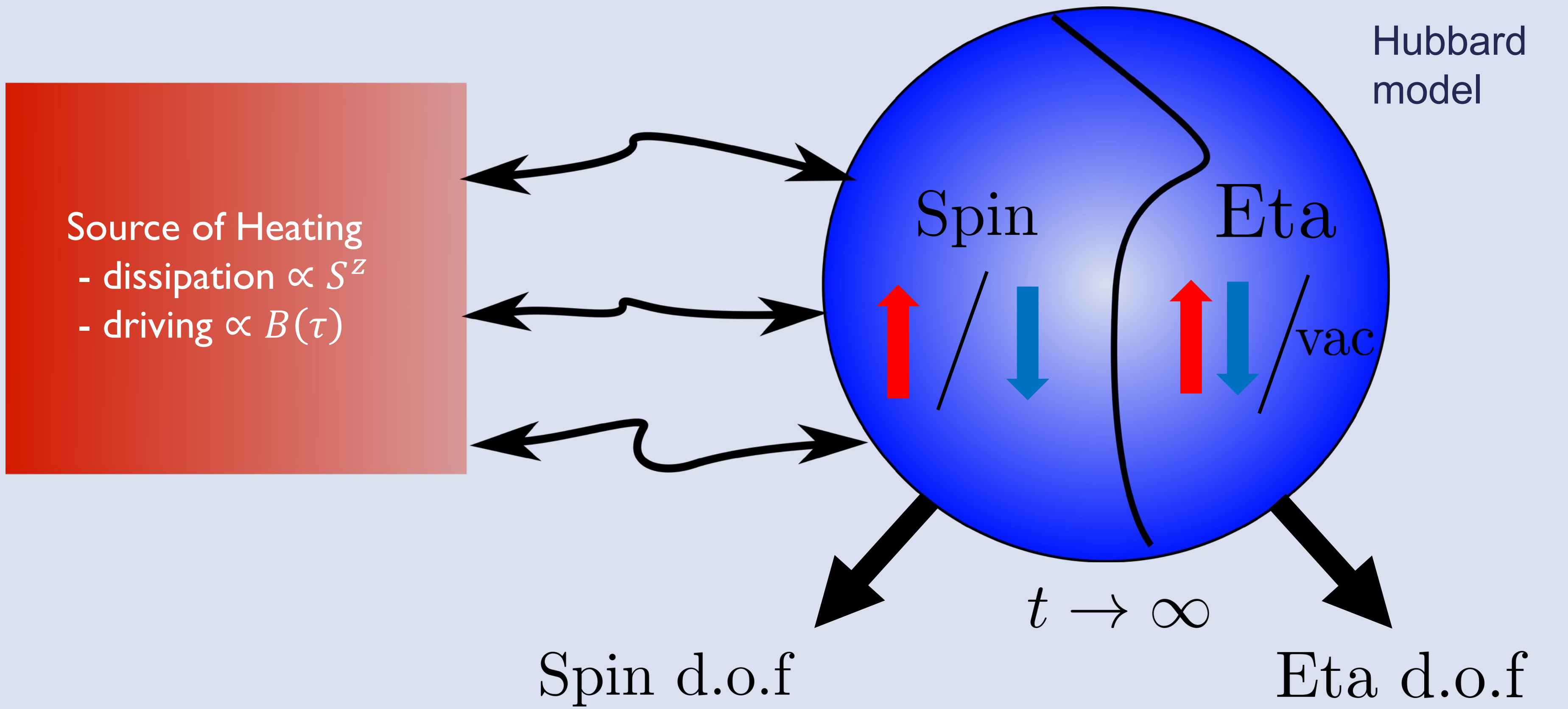


- High energy
- Off-diagonal long-range order
- Superconducting

C. N. Yang, PRL ***63****, 2144 (1989)*

- **Competing order**
- Decaying correlations

Spin-heated Hubbard model

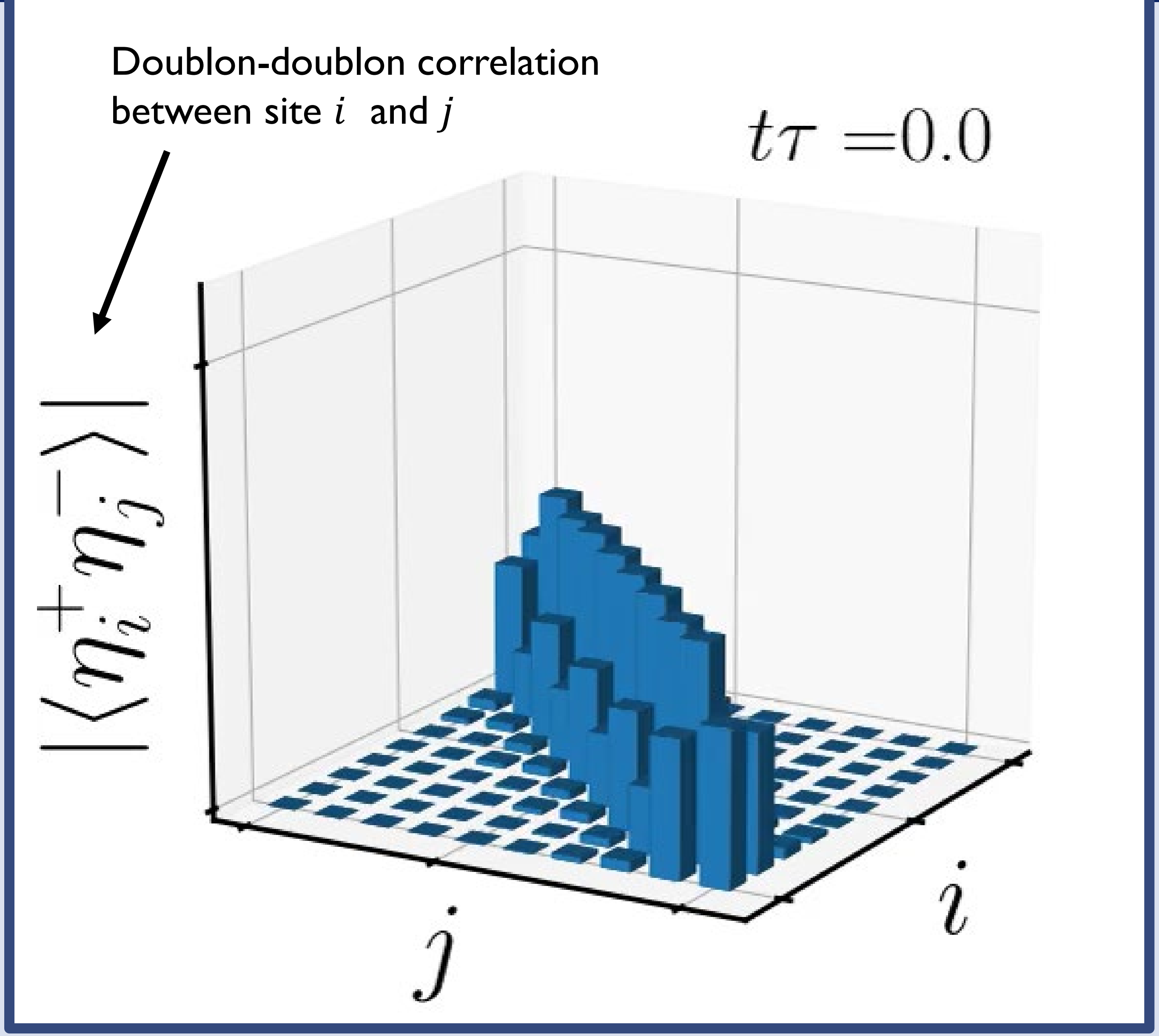


- High Temperature

- Off-diagonal long-range order

J. Tindall, B. Buca, J.R. Coulthard, DJ, Heating-Induced Long-Range η-Pairing in the Hubbard Model, PRL **123**, 030603 (2019)

Spin-heating the Hubbard model from the ground state



- Ground state of the Hubbard model
- Uniform off-diagonal correlations

- η-pairing order
- Superconducting correlations

- No off-diagonal order
- Survives indefinitely

Parameters:
$U = t$

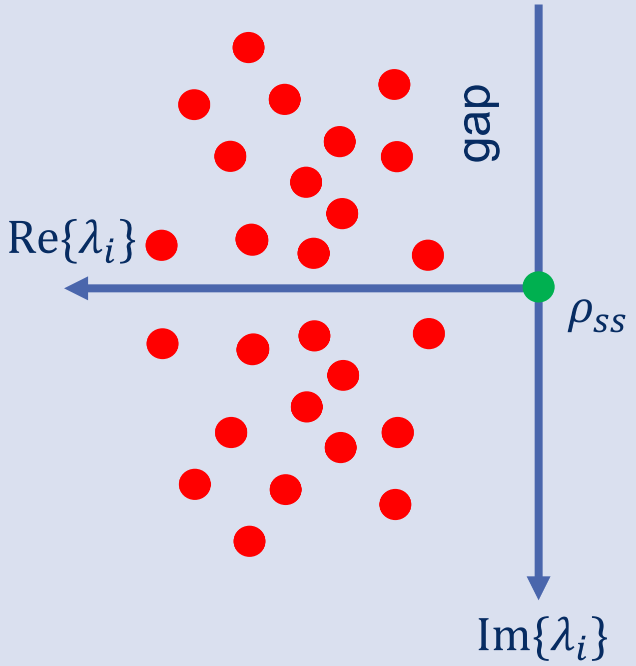
Bath coupling to the spin sector, $B = 0$

- Conserved quantities are $N_\downarrow$, $N_\uparrow$, and $\eta^+\eta^-$
- We assume a stationary state of the form

$$\rho_{ss} \propto \exp(\beta_1 N_\downarrow + \beta_2 N_\uparrow + \beta_3 \eta^+\eta^-)$$

- Lagrange parameters β_i fixing the conserved quantities.
- Since in the stationary state ρ_{ss} we have $[\rho_{ss}, P_{i,j}] = 0$, where $P_{i,j}$ swaps two lattice sites we can show that

$$Tr\{\rho_{ss}, \eta_i^+\eta_{i+j}^-\} = \text{const.}$$

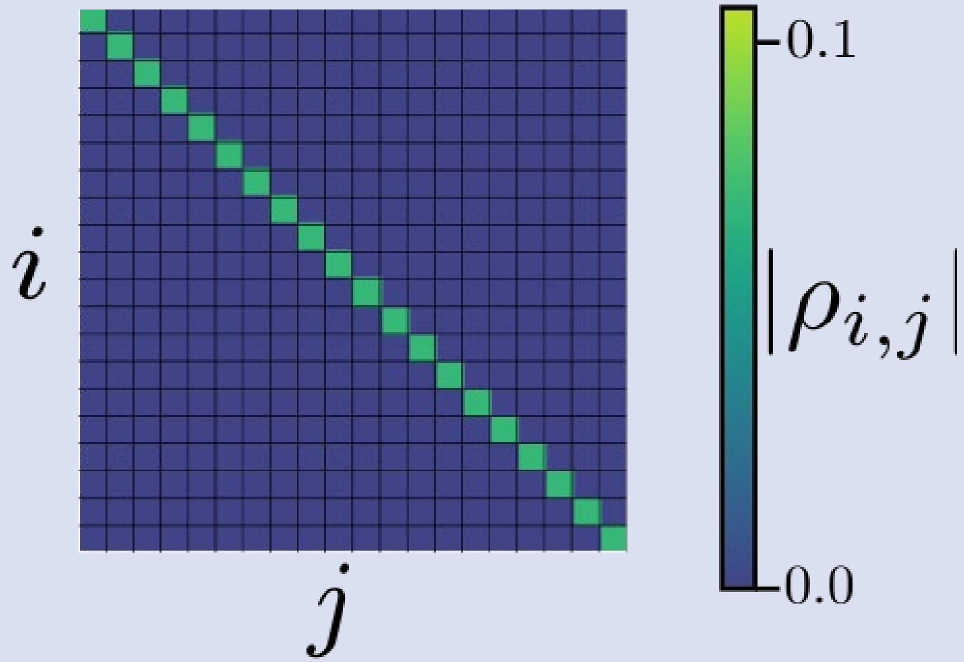


J. Tindall, B. Buca, J.R. Coulthard, DJ, Heating-Induced Long-Range η-Pairing in the Hubbard Model, PRL **123**, 030603 (2019)

Spin- and η-projections

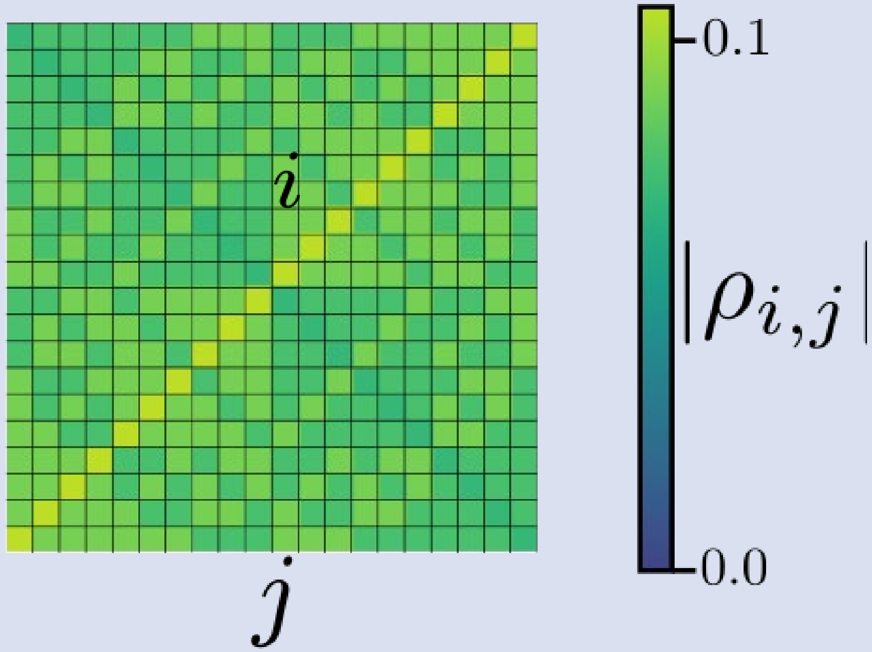
Project into Spin d.o.f.

- Diagonal
- Classical
- Infinite Temperature



Project into Eta d.o.f.

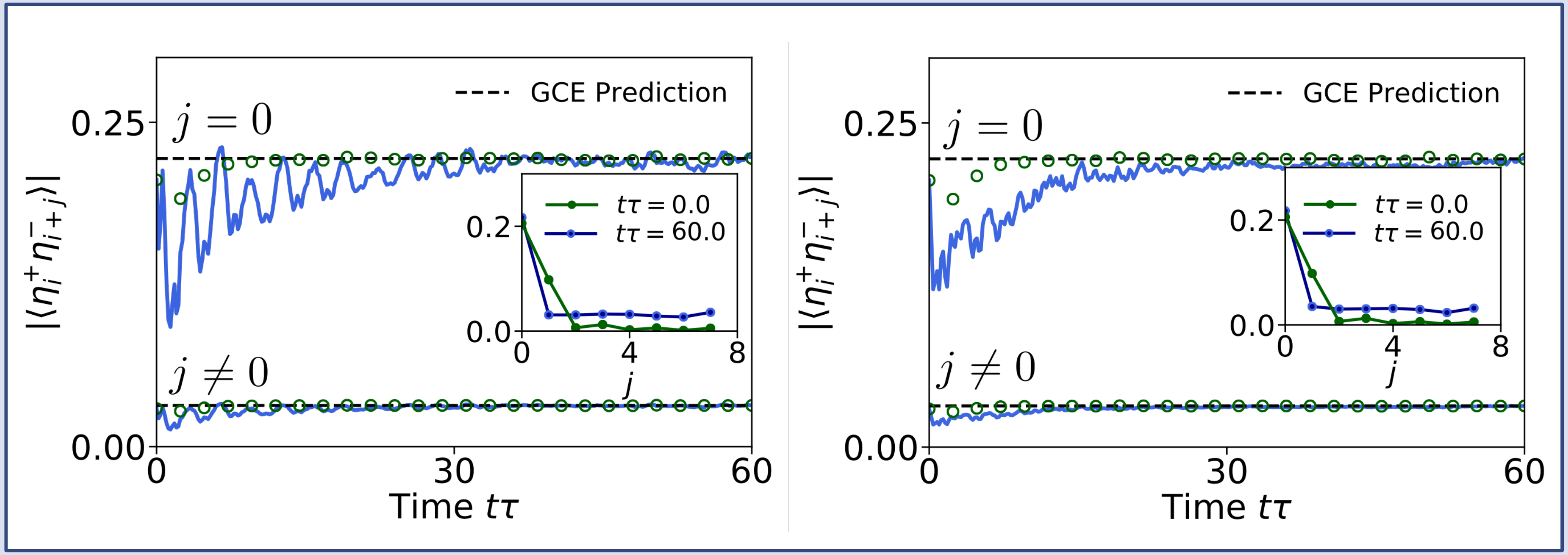
- Off-Diagonal Order
- Entangled
- Low Temperature



Spin driven Hubbard model $B(\tau)$

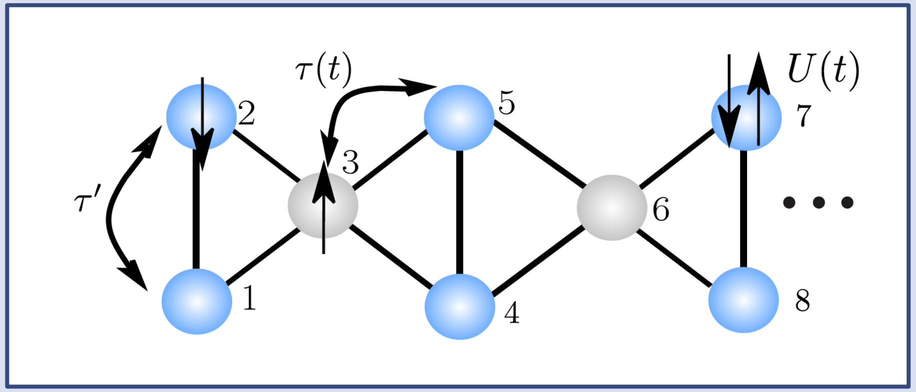
- Strongly drive the isolated system in the spin sector with $B(\tau) = V\cos(\Omega\tau)$

$$H_{\mathrm{Hub}} = -t \sum_{\langle i,j \rangle,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. \right) + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i} n_i + B(\tau) \sum_{i} f_i S_i^z$$



J. Tindall, B. Buca, J.R. Coulthard, DJ, Heating-Induced Long-Range η-Pairing in the Hubbard Model, PRL **123**, 030603 (2019)

Triangular lattice – no η-symmetry, driven hopping

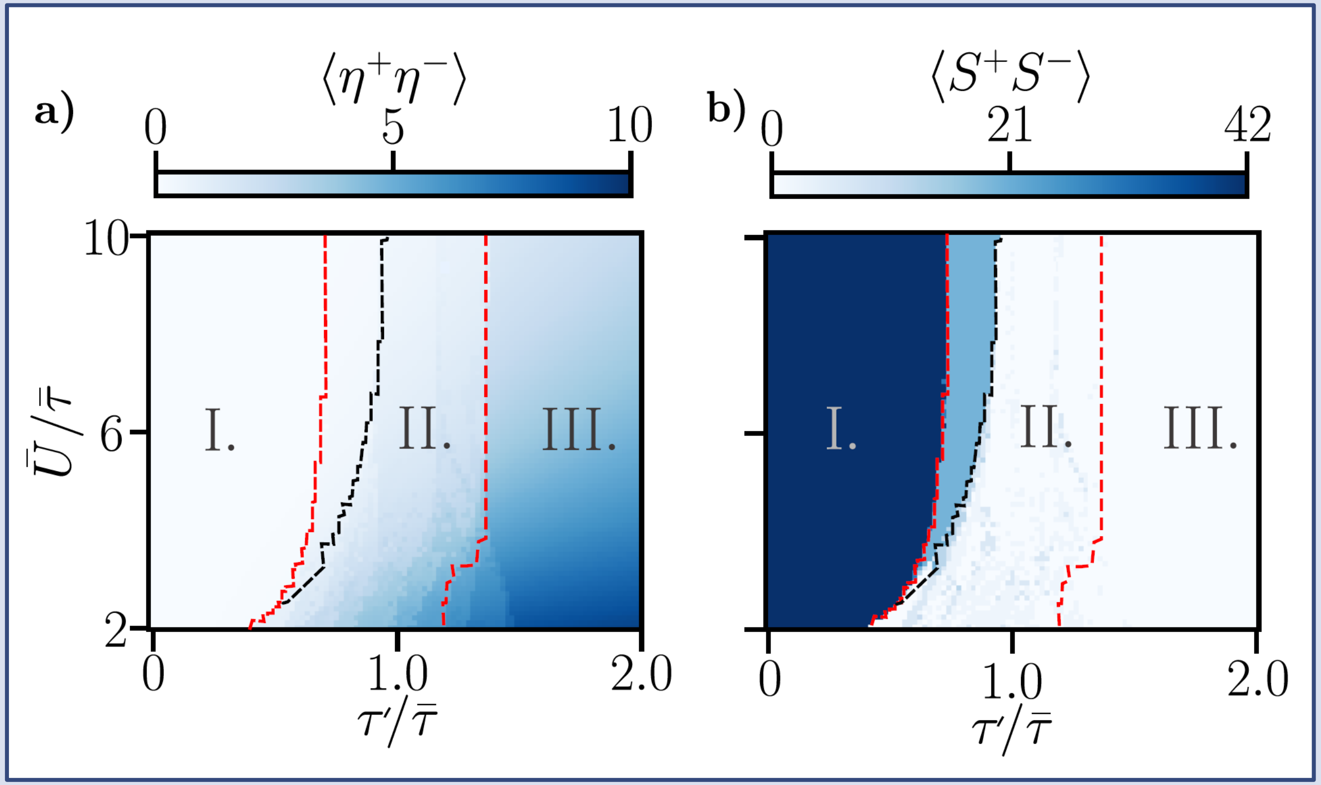


Phase I. for small τ' is a spin wave condensate

$$|\psi\rangle \propto S^{+}|vac\rangle$$

The vertical hopping operator acts as a annihilation operator

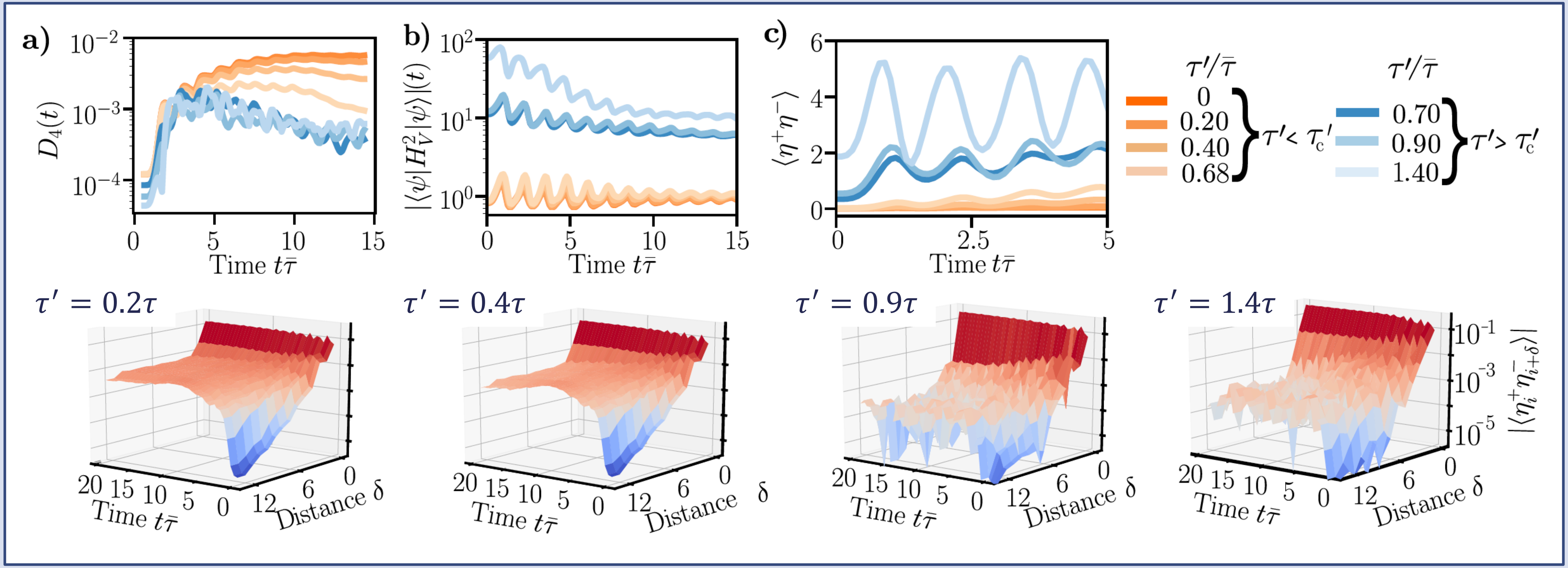
→ shuts down excitation pathway



Frustration causes approximate conservation of η pairs

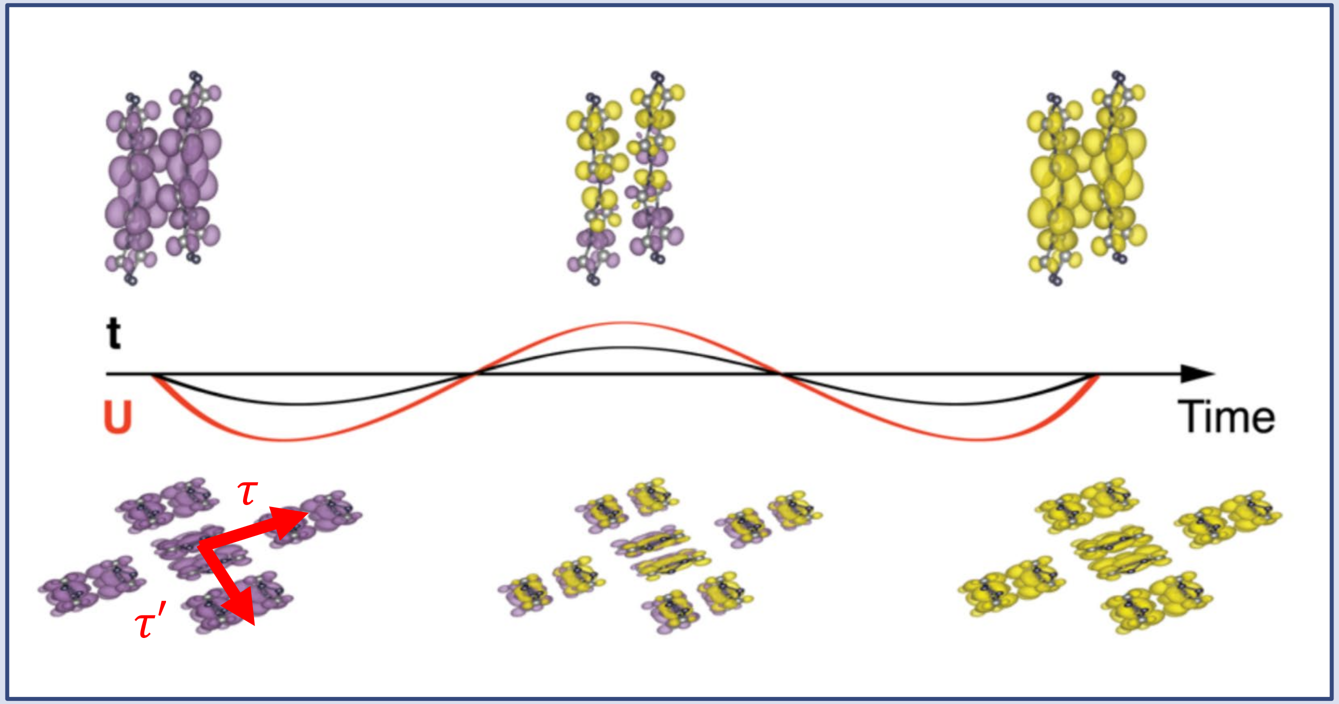
J. Tindall, F. Schlawin, M. Buzzi, D. Nicoletti, J.R. Coulthard, H. Gao, A. Cavalleri, M. Sentef and D. Jaksch, Phys. Rev. Lett. **125**, 137001 (2020).

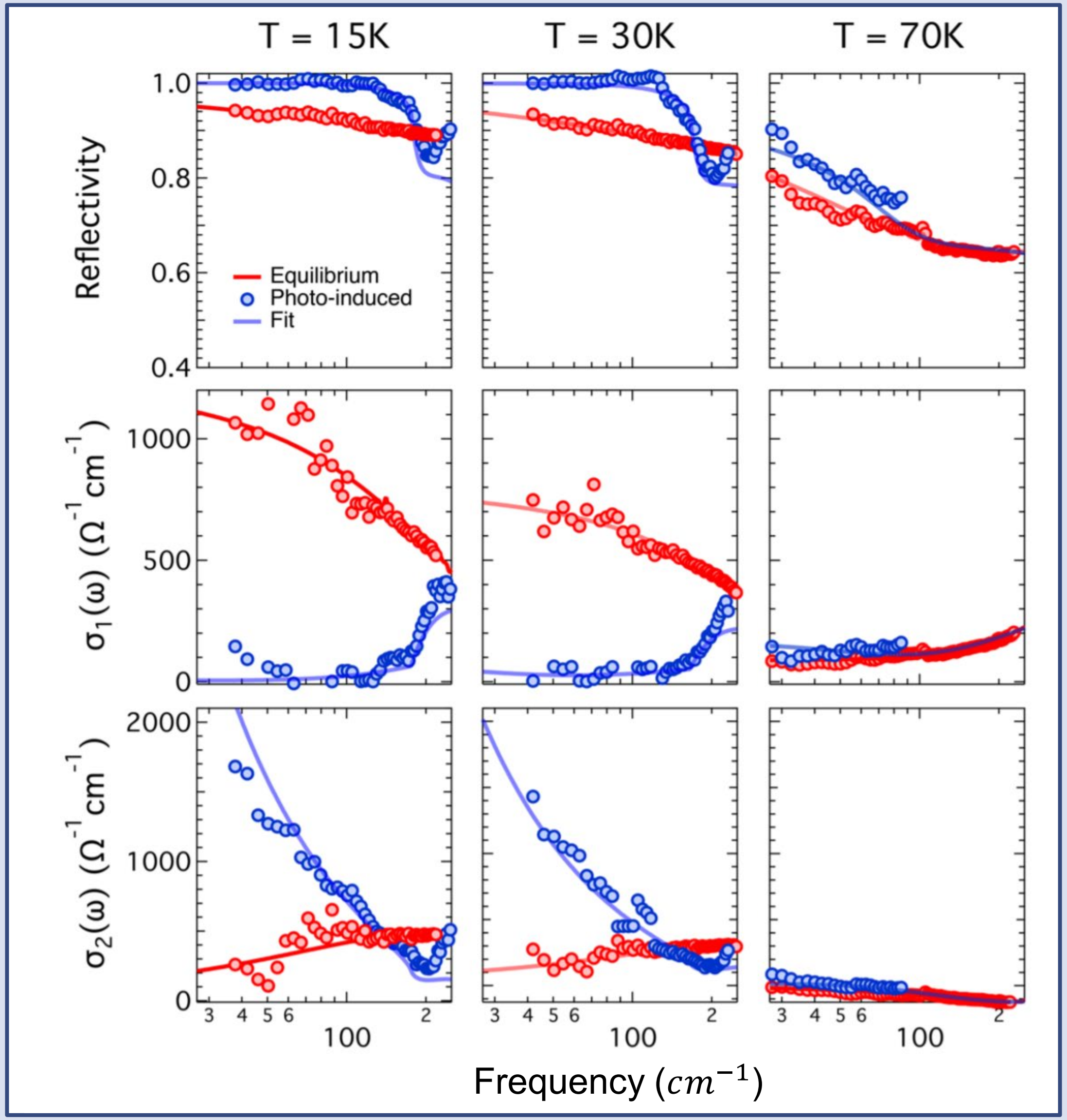
Driving induced η-pairing



In phase I. for $\tau' < \tau_c'$ the number of η pairs is approximately conserved leading to long-range η order

Driven κ-(BEDTTTF)2Cu[N(CN)2]Br





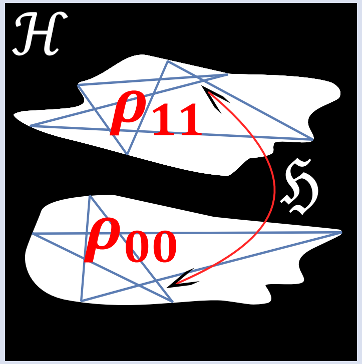
Driving a phonon induces a colossal increase in conductivity up to the characteristic coherence temperature of $T \approx 50K$

Two theorems on the Hubbard model, Elliott H. Lieb
Phys. Rev. Lett. **62**, 1201 (1989)

→ Connection between driving induced pairs and lattice geometry

II. Coupling to the Charge Sector

Bath coupling to the η sector



- We allow a trapping potential $\sum_i \epsilon_i n_i$
- The stationary states are given by

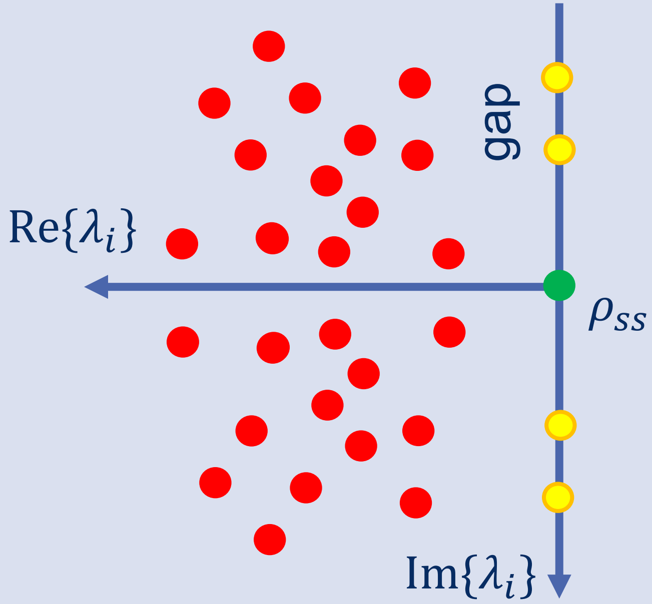
$$\rho_{ss} \propto \exp(\beta_0 N + \beta_1(S^+S^-) + \beta_2 S^z)$$

- Initial states that are superpositions of S^z contain "mixed coherences" of the form (with integer $m, n > 0$)

$$\rho_{nm} = (S^+)^n \rho_{ss} (S^-)^m$$

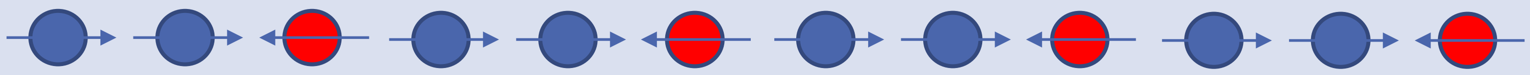
- For $\mathrm{B} \neq 0$ these evolve in a non-stationary way according to

$$\mathcal{L}\rho_{nm} = \mathrm{iB}(m-n)\rho_{nm}$$

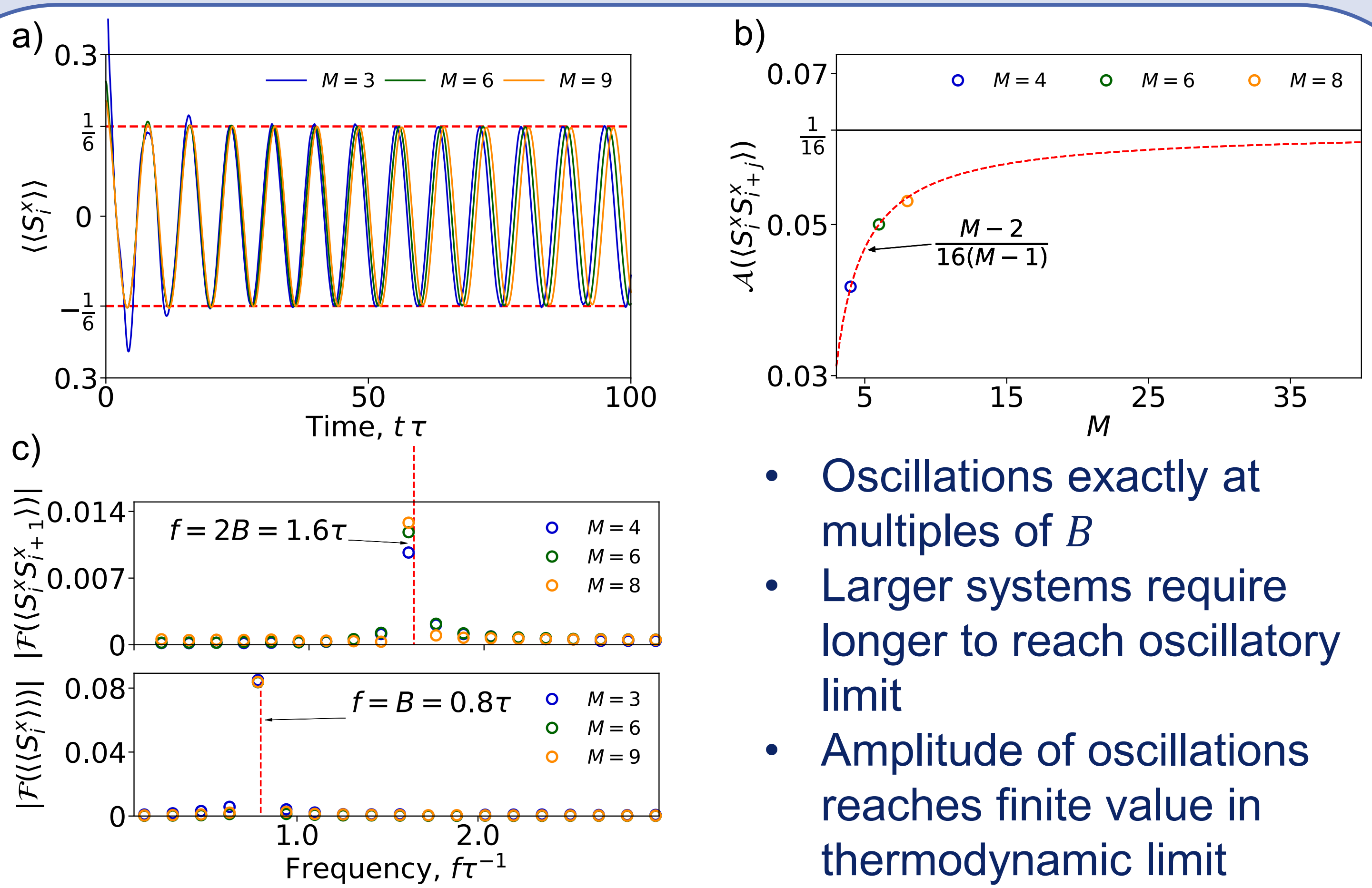


B. Buca, J. Tindall, and D. Jaksch, Nature Communications **10,** 1730 (2019).

Bulk averaged Hubbard dynamics

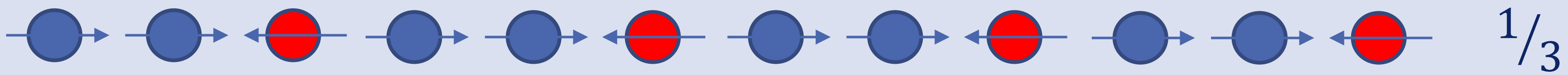


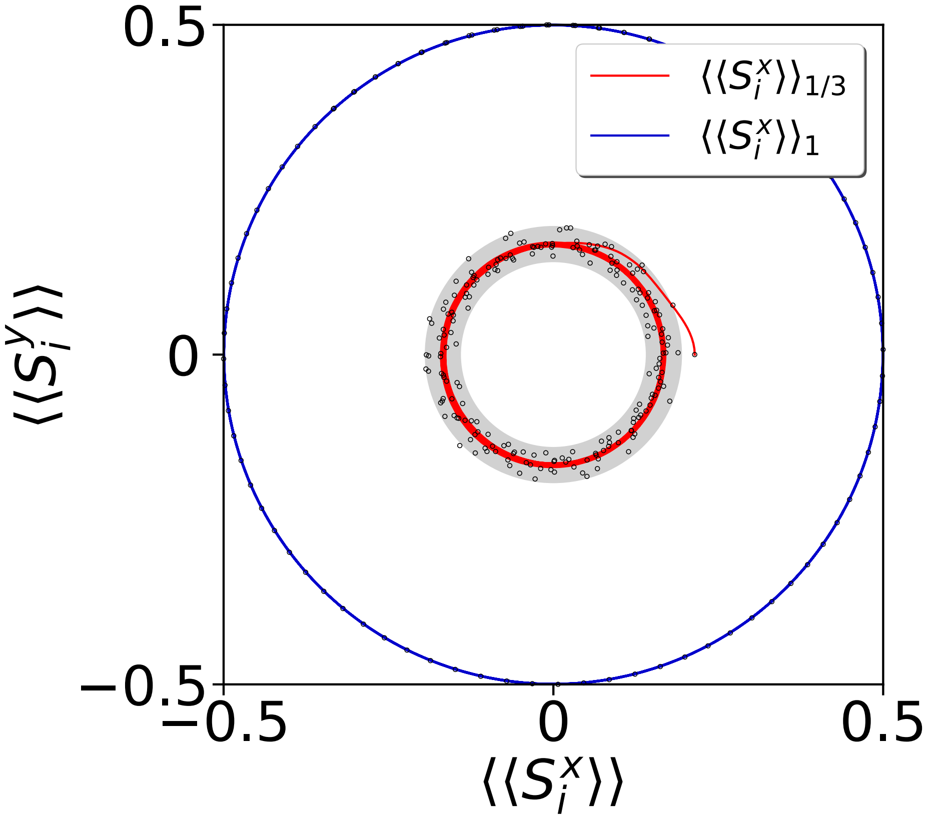
$^1/_3$



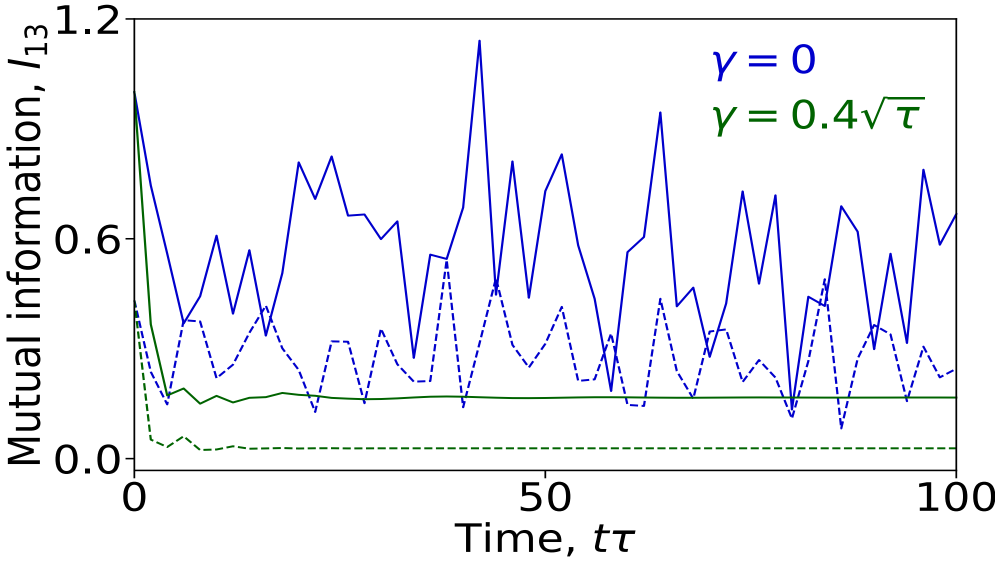
- Oscillations exactly at multiples of B
- Larger systems require longer to reach oscillatory limit
- Amplitude of oscillations reaches finite value in thermodynamic limit

Quantum trajectory simulation



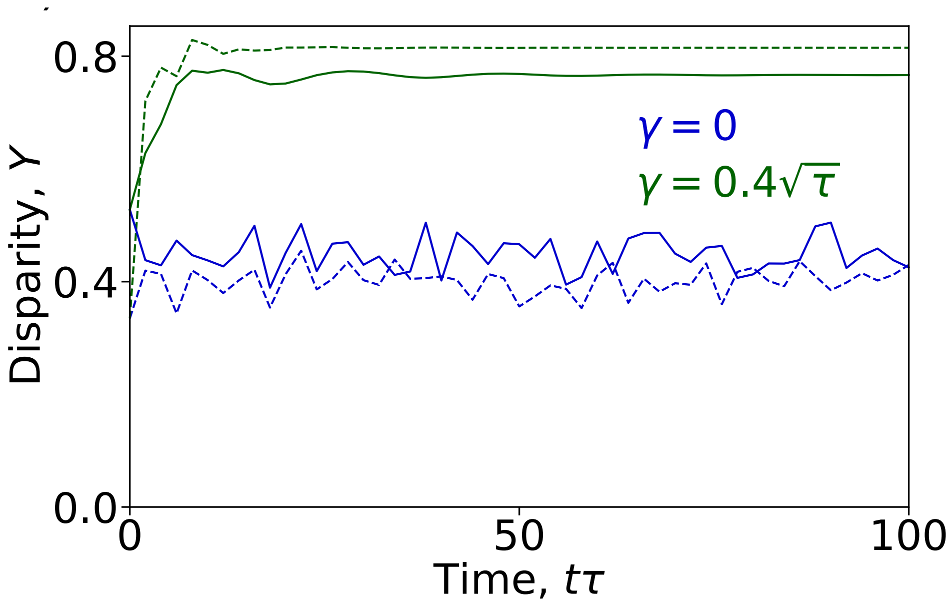


Complexity?



Mutual information:

$$I_{ij} = \frac{1}{2}\left(\mathcal{S}_i + \mathcal{S}_j - \mathcal{S}_{ij}\right)$$



Disparity:

$$Y_i = \frac{\sum_{j=1}^{M} (I_{ij})^2}{\left(\sum_{j=1}^{M} I_{ij}\right)^2}$$

Here $\mathcal{S}_i = \mathrm{tr}(\rho_i \log \rho_i)$, $\mathcal{S}_{ij} = \mathrm{tr}(\rho_{ij} \log \rho_{ij})$, $\rho_i = tr_{k\neq i}\, \rho$ and $\rho_{i,j} = tr_{k\neq i,j}\, \rho$

Limit cycles

- Density operator in the long-time limit

$$\lim_{t\to\infty} \rho(t) = \sum_{n\geq m} C_{n,m}\left(e^{iB(n-m)t}\rho_{nm} + h.c.\right),$$

- where the C_{nm} are determined by the initial state
- An observable $X = \prod_n X_n$ that is a product of M operators acting on individual sites will evolve according to

$$\lim_{t\to\infty} \langle X\rangle(t) = \sum_{n\geq m} D_{nm}\cos(B(m-n)t) + const.$$

- with $D_{nm} = 2C_{nm}\,\mathrm{Tr}\{\rho_{nm}X\}$

→ coherent non-decaying limit cycles of X

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Synchronization

- We consider perturbation $B \to B_n = \bar{B} + \delta B_n$ with

$$\epsilon = \frac{\overline{\delta B_n}}{\bar{B}} \ll 1$$

- and expand $\mathcal{L}$ and ρ in powers of ϵ

$$\mathcal{L} = \mathcal{L}^{(0)} + \epsilon \mathcal{L}^{(1)} + \cdots$$
$$\rho = \rho^{(0)} + \epsilon \rho^{(1)} + \cdots$$
$$\lambda = \lambda^{(0)} + \epsilon \lambda^{(1)} + \cdots$$

- For translationally invariant eigenmodes we find

$$\lambda^{(1)} = \mathrm{Tr}\left\{ \left(\rho^{(0)}\right)^{\dagger} \mathcal{L}^{(1)} \rho^{(0)} \right\} = 0$$

- and otherwise $\lambda^{(1)}$ is purely imaginary

→ limit cycles of X are stable to lowest order

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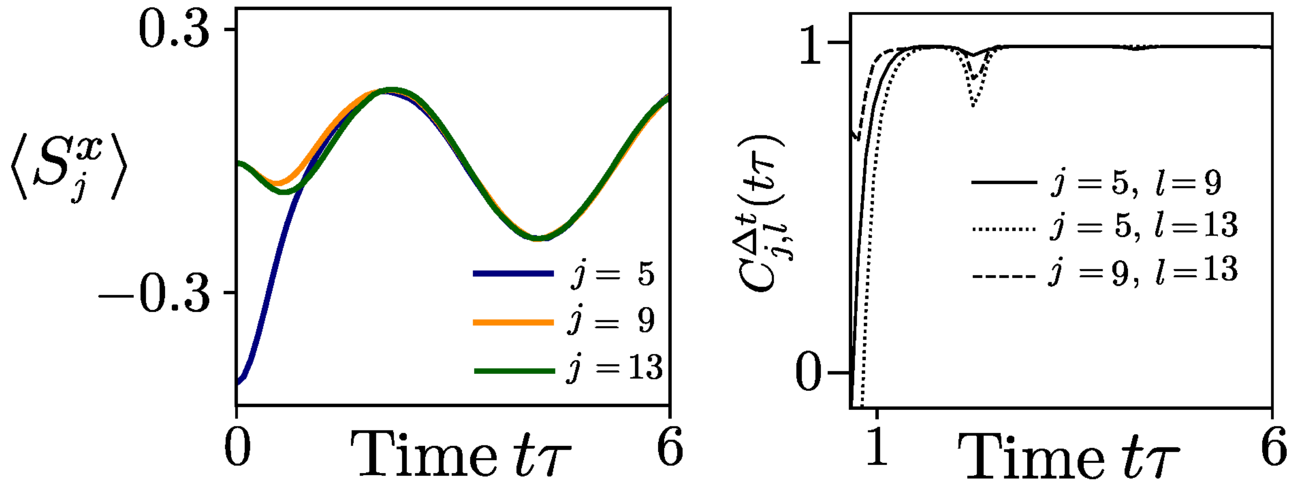
Numerical results

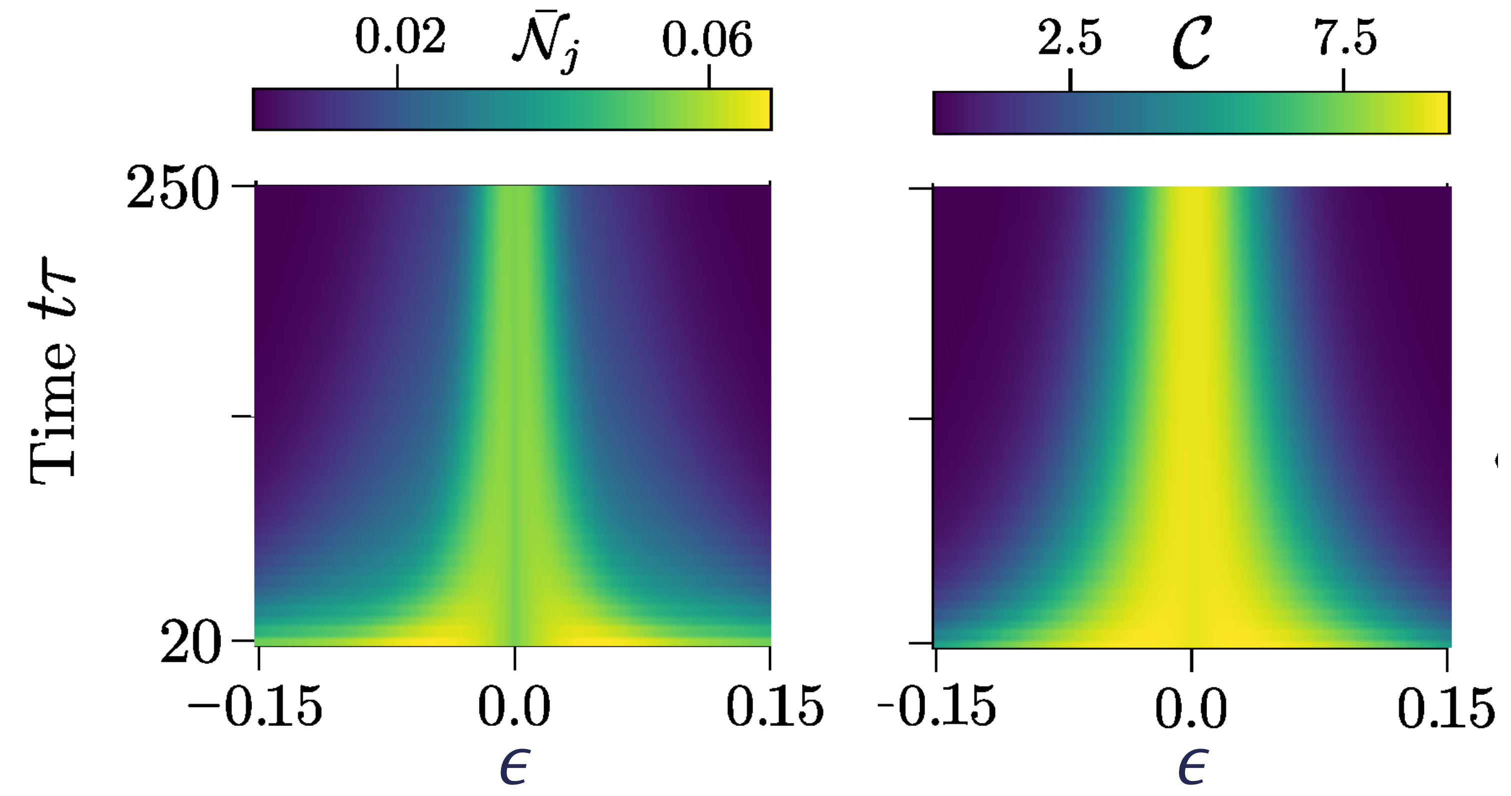


- Observables S_j^x and Pearson correlation coefficient (rolling time window)

$$C_{j,l}^{\Delta t}(\tau) = \frac{\int_{\tau}^{\tau+\Delta t} \left(S_j^x(\tau') - \bar{S}_j^x\right)\left(S_l^x(\tau') - \bar{S}_l^x\right)\mathrm{d}\tau'}{\sqrt{\int_{\tau}^{\tau+\Delta t} \left(S_j^x(\tau') - \bar{S}_j^x\right)^2 \mathrm{d}t' \int_{\tau}^{\tau+\Delta t} \left(S_l^x(\tau') - \bar{S}_l^x\right)^2 \mathrm{d}\tau'}}$$

with $N = 15$, $U = t$, $\bar{B} = 1.5t$, $\epsilon = 0.1t$.

Synchronization witnesses



- Negativity (one site entangled with the others)

$$\mathcal{N}_j(\rho) = \frac{1}{2}\left(\|\rho^{T_j}\| - 1\right)$$

- Off-diagonal coherences

$$C = \sum_{i \neq j} |\rho_{i,j}|$$

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  - Frank Schlawin → Hamburg
  - Carlos Sanchez → Madrid
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  http://www.physics.ox.ac.uk/groups/qubit/

DESOEQ Q-MAC CCDT QSUM Quantum Science with Ultracold Molecules NQIT Networked Quantum Information Technologies EPSRC

Dissipation induced non-stationary complex quantum dynamics

The assumption that physical systems relax to a stationary state in the long-time limit underpins statistical physics and much of our intuitive understanding of scientific phenomena. For isolated systems, this follows from the eigenstate thermalization hypothesis. When an environment is present the expectation is that all of phase space is explored, eventually leading to stationarity.

In this talk, we will identify and discuss simple and generic conditions for dissipation to prevent a quantum many-body system from ever reaching a stationary state [1]. We go beyond dissipative quantum state engineering approaches towards controllable long-time non-stationary dynamics typically associated with macroscopic complex systems. The resulting coherent and oscillatory evolution constitutes a dissipative version of a quantum time-crystal.

We will show how such dissipative dynamics can be engineered and studied with fermionic ultracold atoms in optical lattices using current technology. We discuss how dissipation leads to long-range quantum coherence, complexity, and η-pairing indicating a superfluid state in these setups [2] and the potential connection to driving induced superconductivity [3].

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